Characterization of the performance of optical amplitude-shift keying–differential phase-shift keying orthogonally modulated signals

Ning Deng, Chun-Kit Chan, and Lian-Kuan Chen

Department of Information Engineering, The Chinese University of Hong Kong, Shatin, New Territories, Hong Kong, China

Received September 3, 2004

We analytically investigate the signal performance of orthogonal modulation with slow amplitude-shift keying (ASK) data superimposed upon a high-speed differential phase-shift keying (DPSK) signal. The receiver sensitivities for both the ASK and the demodulated DPSK signals are formulated based on a probability model, and their dependence on the bit rate and the extinction ratio of the ASK data is theoretically investigated. A simple result of analysis of DPSK data performance has been derived. We verified the analytical results by experimental measurements.

OCIS codes: 060.2330, 060.4250, 060.4080, 060.5060

Recently, optical amplitude-shift keying–differential phase-shift keying (ASK–DPSK) orthogonal modulation, in which low-speed ASK data are superimposed onto a high-speed DPSK signal (Fig. 1), has attracted much research attention. It can facilitate control and supervision of optical transport networks and support optical labeling in packet switched networks.

At the receiver, the superimposed ASK signal can be detected directly by a conventional ac-coupled square-law detector, whereas the DPSK signal can be demodulated by passing of the ASK–DPSK signal through an optical delay interferometer (DI) before detection, as shown in Fig. 1. For such orthogonal modulation, the qualities of both the ASK and the DPSK signals strongly depend on the extinction ratio of the ASK signal. In this Letter we theoretically investigate the dependence of the ASK–DPSK receiver’s sensitivity on the extinction ratios as well as on the bit rates of the ASK data. The results are also verified by experimental measurements, with good agreement achieved.

Figure 1 shows a typical temporal waveform of an optical ASK–DPSK signal. Its scalar field can be represented as

\[ E(t) = \sum_m \sum_n \left\{ (\sqrt{r} + (1 - \sqrt{r})a_m)\xi(t - mT_a - t_{a0}) \times \exp[-j\pi d_m \xi(t - nT_d - t_{d0}) - j\omega t - j\phi_0] \right\}, \]

where \( r \) (0 ≤ \( r \) ≤ 1) is the extinction ratio of the ASK signal and is defined as the power ratio of 0 to 1 levels; \( a_m \) is the \( n \)th ASK datum (\( a_{m,n} = 1, 0 \)); \( d_n \) is the \( n \)th precoded DPSK datum (\( d_n = 1, 0 \)); \( \xi(t) \) and \( \zeta(t) \) are the 1-bit baseband waveforms of the ASK and the DPSK signals, respectively, assuming that they are non-return-to-zero rectangular window functions, i.e., \( \xi(t) = \sqrt{P} \) for \( 0 \leq t < T_a \) and 0 otherwise and \( \zeta(t) = 1 \) for \( 0 \leq t < T_d \) and 0 otherwise, where \( P \) is the power of the one level; \( T_a \) and \( T_d \) are the bit periods of the ASK and the DPSK signals, respectively; and \( t_{a0} \) and \( t_{d0} \) are the relative time delays for the ASK and the DPSK signals, respectively.

For an ASK–DPSK signal with \( T_a > T_d \), an optical DI with a relative arm delay of \( T_d \) is used to demodulate the DPSK data. If the coupling ratio of the two couples in the DI is exactly 3 dB, the DI output from one port is

\[ E_{ou}(t) = \exp[-j(\pi/2)](1/2)E(t) + (1/2)E(t - T_d). \]

The output power of the \( n \)th demodulated DPSK bit at its sampling point is

\[ P_n = \frac{1}{2}\left[ (\sqrt{r} + (1 - \sqrt{r})a_{m,n})^2P + \frac{1}{2}(\sqrt{r} + (1 - \sqrt{r})a_{m,n-1})^2P \right. \]
\[ + \frac{1}{2}(\sqrt{r} + (1 - \sqrt{r})a_{m,n})P \times \exp[-j\pi(d_n + d_{n-1})], \]

where \( a_{m,n} \) is the \( n \)th ASK datum that is superimposed upon the \( n \)th DPSK bit at the sampling point. Note that \( a_{m,n} \) and \( a_{m,n-1} \) may stand for the same or adjacent ASK bits. By considering all possible combinations of the ASK bits (represented by \( a_{m,n} \) and \( a_{m,n-1} \)) and of the DPSK bits (represented by \( d_n \)), we tabulated the possible output powers (\( P_n \)) of the \( n \)th demodulated DPSK bit in Table 1, together with their respective probabilities of occurrence (\( \text{prob}_{i} \), \( i \in \{I, II, III, IV\} \)), assuming that 1’s and 0’s are equally probable. The derivation

Fig. 1. Typical waveform of an optical ASK–DPSK signal (left) and its receiver structure: \( \phi_0 \), \( \phi_0 + \pi \), carrier phases of the optical DPSK signal.
Table 1. Output Power of a Demodulated ASK–DPSK Signal

<table>
<thead>
<tr>
<th>Class</th>
<th>ASK</th>
<th>DPSK</th>
<th>Probability of Occurrence</th>
<th>Output Power from DI $P_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0, 0 or 1, 1</td>
<td>-1</td>
<td>$\frac{1}{2} - \frac{1}{4} T_d$</td>
<td>$P_{a} = 0$</td>
</tr>
<tr>
<td>II</td>
<td>0, 0 or 1, 1</td>
<td>-1</td>
<td>$\frac{1}{4} T_d$</td>
<td>$P_{II} = \frac{1}{4} P + \frac{1}{4} P - \frac{1}{2} \sqrt{P}$</td>
</tr>
<tr>
<td>III</td>
<td>0, 0 or 1, 1</td>
<td>+1</td>
<td>$\frac{1}{2} - \frac{1}{4} T_d$</td>
<td>$P_{III} = P$</td>
</tr>
<tr>
<td>IV</td>
<td>0, 0 or 1, 1</td>
<td>+1</td>
<td>$\frac{1}{4} T_d$</td>
<td>$P_{IV} = \frac{1}{4} P + \frac{1}{4} P + \frac{1}{2} \sqrt{P}$</td>
</tr>
<tr>
<td>V</td>
<td>1, 1</td>
<td>+1</td>
<td>$\frac{1}{4} - \frac{1}{8} T_d$</td>
<td>$P_{V} = P$</td>
</tr>
</tbody>
</table>

where $I_D$ is the decision threshold; $R$ is the photodiode’s responsivity; $P_i$ and $\sigma_i$ (for $i \in \{I, II, ..., V\}$) are the output power and the receiver noise in each case, respectively; and erfc() is the complementary error function. In this expanded formula the first two terms correspond to the error probability of bits 0 and the rest of the terms correspond to the error probability of bits 1.

In networking applications of optical ASK–DPSK signals1–3 the payload usually operates at a speed a few times higher than that of the control or the label data, for instance, DPSK payloads at 10 Gbits/s and ASK labels at 2.5 Gbits/s or fewer in optical label switching networks. Thus in our analysis we assume that $T_d/T_a \approx 1/4$. Moreover, with small received power at the receiver, thermal noise dominates; thus we can assume that $\sigma_i$ (for $i \in \{I, II, ..., V\}$) = $rT_d$. By careful inspection we found that, for the extinction ratio of the ASK data ($r$) that lie from 0.63 to 0.95, or for 0.2 dB $\leq$ ER $\leq$ 2 dB (where ER $= -10 \log_{10} \frac{P_{a}}{P_{a}}$) and at the optimal decision threshold $I_D$ obtained below, in Eq. (4) erfc$([I_D - R P_I]/\sqrt{2} \sigma_T)$ is comparable with erfc$([I_D - R P_{II}]/\sqrt{2} \sigma_T)$; both erfc$([R P_{IV} - I_D]/\sqrt{2} \sigma_T)$ and erfc$([R P_{V} - I_D]/\sqrt{2} \sigma_T)$ are smaller than erfc$([R P_{III} - I_D]/\sqrt{2} \sigma_T)$ by a few orders of magnitude. Hence, under these conditions, the first two terms of Eq. (4) can be combined and the fourth and the fifth terms can be neglected. Thus Eq. (4) can be simplified as

$$\text{prob}_{e,d} = \frac{11}{22} \text{erfc} \left( \frac{I_D}{\sqrt{2} \sigma_T} \right) + \frac{11}{42} \text{erfc} \left( \frac{r PR - I_D}{\sqrt{2} \sigma_T} \right).$$

(5)

Letting $\partial \text{prob}_{e,d}/\partial I_D = 0$, we found that the optimal value of $I_D$ can be approximated by $(r PR/2)$, for $\text{prob}_{e,d} \leq 10^{-9}$. At this optimal $I_D$ we have

$$\text{prob}_{e,d} \approx \frac{3}{8} \text{erfc} \left( \frac{Q'_d}{\sqrt{2}} \right), \quad Q'_d = \frac{r PR/2}{\sigma_T}. \quad (6)$$

For $\text{prob}_{e,d} = 10^{-9}$, $Q'_d \approx 5.95$; i.e., $P = 11.9 \sigma_T/(r PR)$. From the power and the probability of each case in Table 1 the average received power of the demodulated DPSK signal is $P_{a} = (r + 1) P/4$. By substituting $P = 11.9 \sigma_T/(r PR)$ into $P_{a}$, we have the average received power at $\text{prob}_{e,d} = 10^{-9}$, i.e., receiver sensitivity $\bar{P}_{\text{rec,d}}$ of the demodulated DPSK signal: $\bar{P}_{\text{rec,d}} = (r + 1)(11.9 \sigma_T)/(4 PR)$. Thus we have

$$\bar{P}_{\text{rec,d}}(\text{dBm}) = \bar{P}_{\text{rec,d}, r=1}(\text{dBm}) + 10 \log_{10}(r + 1)/(2r). \quad (7)$$

Low-speed ASK data with an extinction ratio of $r$, however, can be detected irrespective of phase modulation by direct square-law detection. Thus the receiver sensitivity (at $\text{prob}_{e,a} = 10^{-9}$), $\bar{P}_{\text{rec,a}}$, of the ASK signal is given by

$$\bar{P}_{\text{rec,a}}(\text{dBm}) = \bar{P}_{\text{rec,a}, r=0}(\text{dBm}) + 10 \log_{10}(1 + r)/(1 - r). \quad (8)$$
Using Eq. (7), we have drawn (Fig. 2, solid curve) the receiver sensitivity of the 10-Gbit/s (Gb/s) demodulated DPSK data, $\bar{P}_{\text{rec,d}}$, versus various extinction ratios [dB] of the ASK data, assuming that $\bar{P}_{\text{rec,d},r=1}$ was $-19.2$ dBm at $\text{ER}_{\text{dB}}=0$ (i.e., no ASK data, $r=1$). We also numerically computed the relation between the receiver sensitivity of the DPSK data and the extinction ratio of the ASK data according to Eq. (4). We found that the simplified analytical curve (solid curve) almost coincided with the numerically computed curves for different ASK bit rates for $\text{ER}_{\text{dB}} < 3.5$ dB. This finding validates the assumptions and simplifications used in our analytical model. Furthermore, this range of $\text{ER}_{\text{dB}}$ values is a practical case, as the extinction ratio of ASK data is usually kept small in ASK–DPSK applications to minimize possible degradation in performance of the high-speed DPSK payload. When the extinction ratio of ASK data increased beyond 3.5 dB, the difference between the simplified analytical curve and the numerically solved curves started to increase, owing to the nonnegligible deviation induced in simplification from Eq. (4) to relation (5) for this range of ASK extinction ratios. Moreover, we noted that the demodulated DPSK signals had almost the same performance, even at different ASK bit rates, when $\text{ER}_{\text{dB}} < 3.5$ dB. For $\text{ER}_{\text{dB}} > 3.5$ dB, the demodulated DPSK signal had slightly worse receiver sensitivity when the superimposed ASK signal was at higher bit rates.

We performed experimental measurements to verify the analytical results. The DPSK bit rate was chosen to be 10 Gbits/s, and the superimposed ASK data were at bit rates of 1 and 1.25 Gbits/s. A p-i-n photodiode with a matching electrical bandwidth was used to detect the ASK signals directly. The receiver sensitivities of 1- and 1.25-Gbit/s ASK signals with $r=0$ were measured to be $-26.1$ and $-25.75$ dBm, respectively. Figure 3 depicts their measured receiver sensitivities (exp) at other extinction ratios of the ASK data. By using Eq. (8), we also plotted their analytical receiver sensitivities (ana); Fig. 3 shows good agreement of theory with the experimental results.

To measure the demodulated DPSK sensitivity we first fed the ASK–DPSK signal into an optical DI before it was detected by a 10-Gbit/s p-i-n diode. We measured the demodulated signal power at a bit-error rate of $10^{-9}$ before it was fed into the photodiode. With the measured receiver sensitivity of $-19.2$ dBm for the demodulated 10-Gbit/s DPSK signal without any ASK signal (i.e., $r=1$), we plotted the solid curve in Fig. 3 by using Eq. (7). The experimental sensitivities are also shown in Fig. 3 by open squares, circles, and triangles for superimposed ASK signals at 2.5, 1.25, and 1 Gbits/s, respectively. Experimental results showed that with the extinction ratio of the ASK data in the range 0.8–2.1 dB the measurements agreed well with the simplified analytical results and the demodulated DPSK sensitivity had nearly no dependence on the ASK bit rate.

In conclusion, we analytically investigated the signal performance of an ASK–DPSK orthogonal modulation format based on a probability model, assuming Gaussian noise characteristics. The calculations imply an analytical relationship between the ASK–DPSK receiver sensitivity and the extinction ratio of the ASK data. The results obtained have been proved to agree with the numerical results as well as with the experimental measurements.

This project was partially supported by a research grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (project CUHK4240/04E). N. Deng’s e-mail address is ndeng@ieee.org.

References