Unequal Error Protection Based on OFDM
and Its Application in Digital Audio Transmission

Keang-Po Ho

Department of Informational Engineering, The Chinese University of Hong Kong, Shatin, NT, Hong Kong
Tel: +852 2609 8357, Fax: +852 2603 5032, E-mail: kpho@ie.cuhk.edu.hk

Abstract:
Orthogonal frequency division multiplexing (OFDM) can be used naturally and effectively to provide unequal error protection by properly allocating power and assigning constellation to each individual subchannel. The principle and algorithm of using power allocation is first reviewed and applied to digital audio as an example. MPEG-1 audio is transmitted through an OFDM system. It is found that OFDM with unequal error protection can achieve 4 to 6 dB of coding gain over a single-carrier system having the same bandwidth.

1. Introduction

Conventionally, source and channel codes are designed separately and then cascaded together. Provided with an infinite degree of complexity, source and channel coding can be separated without sacrificing fidelity [1]. However, when many source coding techniques are applied, some coded bits (e.g., the most significant bit of a scalar-quantized signal) are far more important than others. In practice, unequal error protection by some means, for example, those in cellular systems [2], is provided to improve the system performance.

Power allocation, especially suitable for a multicarrier modulation system [3]-[5] shown in Fig. 1, distributes the available quota of transmission power to different bits according to their importance in order to provide the best output quality at the receiver output. Power allocation was described by Bedrosian [6] and Sundberg [7] for pulse-coded modulation, [8]-[12] for vector quantizers, and Ho and Kahn [13] for image transmission. It was shown that power allocation can be applied for multicarrier modulation or its generalized version [8]-[9], [11], [13].

Orthogonal Frequency Division Multiplexing (OFDM), a type of multicarrier modulation, is the standard modulation scheme for digital audio broadcasting (DAB) [14]-[15] and is also proposed for digital television [16]. In this paper, we demonstrate by simulation that power allocation in OFDM can be used to provide unequal error protection for gradual quality degradation as the channel becoming more noisy. While the design of the whole working system is not provided, some numerical results show significant improvement of OFDM over single-carrier systems.

In the remaining part of this paper, we will first review method and algorithm on power allocation in OFDM system for unequal error protection provisioning. We will also review digital audio compression scheme. The application of power allocation to

![OFDM System Diagram](Fig. 1 OFDM system, where $E_i$ is the power transmitted in the $i$th carrier, $f_i$ is the center frequency of the $i$th carrier, $n_i$ is the number of bits per symbol of the $i$th carrier, and $B_i$ is the bandwidth of the $i$th carrier.)
digital audio is described later with numerical results provided by extensive simulation.

2. Power Allocation in OFDM

The schematic diagram of an OFDM system is shown in Fig. 1. Each subchannel can utilize different power, modulation, channel encoding, and bandwidth, so that different degrees of error protection may be provided for different bits according to their importance to immune distortion at the output of the receiver. OFDM is a multichannel modulation system and all regular [3]-[5] or “generalized” [8] multichannel modulation schemes can be used to substitute the OFDM scheme.

To illustrate the power allocation algorithm in OFDM, for a simple example, we assume that the source is quantized by a uniformly spaced scalar quantizer, and that different bits are transmitted via different OFDM subchannels. Assume that $\Delta$ is the normalized uniform level spacing for a source having zero mean and unit variance. For a source having a variance of $\sigma_s^2$, the channel-induced distortion for an $n$-bit uniformly quantized signal is approximately equal to [6][17]

$$e_n^2 / \sigma_s^2 = \Delta^2 \sum_{i=0}^{n-1} P_{b_i} 2^{2i}.$$  

where $P_{b_i}$ is the BER of the $i$th bit. The exact channel-induced distortion for uniformly quantized signal can be found in [17]. Note that the importance of the $i$th bit can be defined as $w_i = 2^{2i} \Delta^2 \sigma_s^2$ such that the channel-induced distortion is simply a weighted summation of BERs, i.e.,

$$e_n^2 = \sum_{i=0}^{n-1} w_i P_{b_i}.$$  

(2)

We will use an $n$ subchannel (or a multiple of $n$ subchannels) OFDM system to transmit an $n$-bit signal, assigning different bits to different subchannels in consecutive order. The BER $P_{b_i}$ is a function of $E_i$, the power in the $i$th subchannel, i.e., $P_{b_i} = P_i(E_i)$. The sum of these powers is equal to the total power $E_T$. Since the intrinsic distortion due to quantization (or compression) is fixed, and is almost uncorrelated with channel-induced distortion [17], the power allocation problem is to choose the subchannel powers $E_i$ to minimize the channel-induced distortion:

$$e_n^2 = \min_{E_i \geq 0} \left\{ \sum_{i=0}^{n-1} w_i P_i(E_i) \right\}.$$  

The BER function $P_i(\cdot)$ may be different for each subchannel, depending on the choice of modulation and channel coding. This problem of (3) and (4) can be solved numerically by introducing a Lagrange multiplier [6]-[13]. Using 16-QAM modulation for a Gaussian channel and 8-bit quantizer for a Gaussian source, Fig. 2 shows both the signal-to-channel-induced-distortion ratio $\sigma_s^2 / e_n^2$, and the overall signal-to-distortion ratio (S/D) $\sigma_s^2 / (e_n^2 + \sigma_q^2)$ (where $\sigma_q^2$ is the mean-squared quantization-induced distortion), as a function of the average SNR of the channel.

As inferred from (2) and (3), the only and most important information the source coder requires to provide for power allocation is the importance of each bit, $w_i$. Also, only the relative values of the importance are required in the algorithm, absolute values are not essential. While the importance of bit in an uniform quantizer can be find easily, the importance of bit for data compression in general is more difficult to find.

3. MPEG-1 Audio Compression

OFDM is being used for DAB standard in Europe [14]-[15] in which unequal error protection
is provided by rated-punctured convolution code [18]. As shown in previous section, OFDM can provide unequal error protection naturally by power allocation. Since DAB for other countries is still not yet standardized, this paper exploits the possibility of using OFDM with natural error protection for audio transmission. In this section, we review MPEG-1 audio compression scheme [19] and estimate the importance of bit in the data stream. Unequal error protection is not limited to MPEG-1 audio which is used here just for its simplicity.

The MPEG audio bases on probably the most well-know audio coding algorithm: MUSICAM (Masking-pattern Universal Subband Integrated Coding and Multiplexing) [20]-[21]. It may also conceptually be the simplest audio coder. The audio is first separated by using a 32-band uniform filter bank, obtained by modulation of a prototype low-pass filter. In parallel to the filter bank, a fast Fourier transform is used for spectral estimation. Based on the power spectrum of the audio signal, a masking curve is calculated according to a perceptual model for human hearing mechanism: a strong tone in one frequency can mask the signal at nearby frequencies. Quantization noise is then allocated in various sub-bands according to the masking function. This allocation is done on a small block of subband samples (typically 12). The maximum value within a block, call scale factor, and the quantization step, based on masking, are calculated for each block. They are transmitted as side information, together with the quantization samples. MUSICAM does not use entropy coding, the quantized values are sent (almost) directly.

To utilize power allocation for unequal error protection, the source coder must provide the importance of each bit to the channel coder. While those informations are not available directly from the audio coder, the approximated numerical importance of each bit can readily be derived from the coded data stream.

For uniform quantizer, based on mean-squared distortion, \( w_i = 2^{2i} \Delta \sigma^2 \), where \( \Delta \sigma^2 \) is the quantization level. In MUSICAM, the quantization level is approximately equal to \( (\text{scale factor})/2^n \), where \( n \) is the number of allocated bit for the subband. The approximate importance is

\[
w_i = 2^{-2(n-i)} (\text{scale factor})^2 . \tag{5}
\]

The importance defined by (5) uses the mean-square difference without taking into account the perceptual masking pattern. The remaining part of this paper will use the mean-squared distortion based importance to characterize the audio performance for simplicity. If the perceptual masking pattern is taking into account, the importance may be defined as \( w_i = 2^{2i} \) by normalized all quantization level to be approximately perceptually equal.

4. Numerical Results

Audio transmitted over an additive white Gaussian noise (AWGN) channel is considered as a numerical example. May not be a practical model for audio transmission system, an AWGN channel is considered for its simplicity. A long (32 kb/s, 6 mins) stereo linear 16-bit audio signal is first encoded using a MPEG-1 shareware based encoder [22]. The compressed data stream is passed through a single- or multicarrier channel and then a shareware based decoder [22]. For simplicity, we assume that the important side-information (bit-allocation table, scale-index, sync-bit, etc.) is transmitted through another error-free channel (for example, with strong error protection or robust modulation). The numerical results compare the transmission of subband audio samples only. Also, only MPEG-1 layer-I audio is used for simplicity.

The power allocation algorithm requires the knowledge of channel signal-to-noise ratio (SNR). While it is theoretical interested to know the best achievable performance by assuming that the SNR is given a priori, the channel SNR may not be available at the transmitter, especially for DAB system for a broadcasting channel with many receivers. Unequal error protection can still be provided and optimized for a specific SNR. However, the performance may be degraded for other mismatched channel SNRs. A very simple OFDM system can be used in which all subchannels have the same channel SNR but different modulation constellation.

In the simulation, the compressed audio sequence is transmitted through an uncoded-OFDM system with two QPSK subchannels, two 8-QAM subchannels, one 16-QAM subchannel, and one 32-QAM subchannel for a total of six subchannels (or integer multiply of six subchannels) and 19 bits per OFDM symbol. Two sets of simulation with optimal power allocation (for theoretical interested) and identical channel SNR (more realistic) are con-
ducted for comparison. Other simulations are conducted for a single-carrier system in which all bits are provided with an identical error protection.

To illustrate the power allocation algorithm, Fig. 3 shows the relative importance, BER, and subchannel SNR for a digital audio frame with either optimal or equal subchannel SNR. The bits are better transmitted with different constellations in a greedy rule such that the less importance of a bit, the larger the constellation and the larger the BER. The transmitter requires to sort the importance in descending order and allocates subchannels with simple constellation to important bits first. With the same average subchannel SNR for both cases, power allocation is required for optimal performance but not for the equal-power sub-optimal case. In Fig. 3, the bits in a digital audio frame are not ordered in decreasing importance but have been sorted for convenience for presentation. The importance of bit has a range of seven order. When the subchannel constellation changes from QPSK to 32-QAM, the BER of the subchannel increases from $10^{-5}$ to 0.3. The subchannel SNR is also in general decreasing with the importance of the bit but increases when the subchannel increases to larger constellation. In general, the subchannels carrying important bits are allocated with more power but the subchannels carrying less important bits are allocated with less power for optimal performance.

Fig. 4 shows the output audio signal-to-distortion ratio (S/D) versus the average channel SNR. The audio S/D is defined as the ratio of the power of the original audio samples ($x_{in}$) before encoder to the square of the difference between the original audio samples and the output audio samples ($x_{out}$) at the decoder, i.e. $S/D = E\{x_{in}^2\}/E\{(x_{in} - x_{out})^2\}$. The perceptual model of human being is not taken into account in the evaluation of audio S/D for simplicity.

The digital audio is transmitted through several single- and multicarrier systems in Fig. 4. As expected in OFDM case, the best performance is achieved by the system having optimal power allocation. Although may not be practical in broadcasting system, optimal power allocation in OFDM can achieve a coding gain (in channel SNR) of 1 to 1.5 dB over equal power allocation.

The single-carrier systems can use BPSK, QPSK, 8-QAM, or other modulation. The system with BPSK is obvious to achieve the best performance. The OFDM system uses six subchannels to carry 19 bits per symbol. The BPSK, QPSK, and 8-QAM single-carrier system occur a bandwidth of 3.17, 1.58, and 1.06 times the total bandwidth of the OFDM system. Even with more bandwidth, the single-carrier 8-QAM system is much worse (about 4.5 to 6 dB in channel SNR) than that of OFDM system for the same audio S/D. The single-carrier QPSK is slightly better than the equal-power OFDM system but 1 to 1.5 dB worse than that of optimal power OFDM.

The OFDM system should compare with 8-QAM single-carrier system because of approximately using the same bandwidth. The equal-power OFDM system can achieve a coding gain of about 4.5 dB in channel SNR for the same audio S/D. Compared with QPSK single-carrier system, equal-power OFDM system achieves the same performance in most of the average channel SNR values. However, equal-power OFDM system uses just about 62% the bandwidth of QPSK single-carrier system.

With the same channel SNR, the improvement in audio S/D is very significant using OFDM. The improvement is larger than 20 dB in the region of low channel SNR.

5. Conclusion

OFDM system is used to provide unequal error protection by properly allocating power and assigning constellation to each individual subchannel. The principle and algorithm of using power allocation is first reviewed and applied to digital audio as an example. The channel distortion is approximated as a weighted sum of the subchannel BERs in which the weighting is approximately evaluated for digital audio.

Simulations are conducted in which MPEG-1 audio is transmitted through OFDM or single-carrier systems. It is found that OFDM with unequal error protection can achieve 4 to 6 dB of coding gain for the same audio S/D. For the same channel SNR, significant performance improvement of 20 dB is also found in the output audio S/D, especially for the region with low channel SNR.
Fig. 3 Importance, BER, and SNR of each subchannel or bit in a MPEG-1 audio frame. BER for optimal and equal power allocation for each subchannel are shown for comparison.

Fig. 4 The received audio S/D as a function of average channel SNR with equal or optimal power OFDM transmission, BPSK, QPSK, and 8-QAM single-carrier transmission.
6. References


