Performance Characterization of Optical Orthogonal ASK/DPSK Signals
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Abstract We analytically investigate the performance of optical orthogonal modulation – slow ASK data superimposed on high-speed DPSK signal. The receiver sensitivities for both ASK and demodulated DPSK signals are formulated and are verified by experimental measurements.

Introduction
Recently, optical ASK/DPSK orthogonal modulation, in which the low-speed amplitude shift-keying (ASK) data is superimposed onto the high-speed differential phase shift-keying (DPSK) signal (see Fig. 1), has attracted much attention. It facilitates control/supervision in optical transport networks [1], and optical labeling in packet switched networks [2]. At the receiver, the ASK signal can be directly detected by a square-law detector while the DPSK signal is demodulated by an optical delay interferometer (DI) before detection, as shown in Fig. 1. For such orthogonal modulation, the qualities of both the ASK and the DPSK signals strongly depend on the extinction ratio (ER) of the ASK signal. In this paper, we analytically investigate the dependence of ASK/DPSK receiver sensitivity on the ER as well as the data rates. The results are also verified by experimental measurements.

Theoretical analysis
Fig. 1 shows a typical waveform of an optical orthogonal ASK/DPSK signal. Its scalar field can be represented as

\[
E(t) = \sum_n \sum_m \left[ \sqrt{P} \exp(-j \pi a_n) f(t - m T_d - t_0) \right] \exp(-j \pi d_n g(t - n T_a - t_0)) \]

where \(P\) is the power of one level; \(T_a\) and \(T_d\) are the bit periods of the ASK and the DPSK signals, respectively; \(t_0\) and \(t_0\) are relative time delays, for the ASK and the DPSK signals, respectively.

For the orthogonal ASK/DPSK signal, an optical DI with a relative arm delay of \(T_d\) is used to demodulate the DPSK data. The DI output from one port is:

\[
E_{aw}(t) = e^{-\frac{t}{T_d}} \left[ 1/2 E(t) + 1/2 E(t - T_d) \right]
\]

The output power of the \(n\)th demodulated DPSK bit at its sampling point is:

\[
P_n = \frac{1}{4} \left[ \sqrt{P} + (1 - \sqrt{P}) |a_{n1}| \exp(-j \phi) + \frac{1}{2} \sqrt{P} + (1 - \sqrt{P}) |a_{n2}| \exp(-j \phi) \right] \]

\[
= \frac{1}{4} \left[ \sqrt{P} + (1 - \sqrt{P}) |a_{n1}| \exp(-j \phi) + \frac{1}{2} \sqrt{P} + (1 - \sqrt{P}) |a_{n2}| \exp(-j \phi) \right]
\]

where \(a_{n1}\) is the ASK data being superimposed on the \(n\)th DPSK bit, at the sampling point. By considering all possible combinations of the ASK bits \((a_{n1}, a_{n2})\) and the DPSK bits \((\exp(-j \phi), \exp(j \phi))\), the output powers \(P_n\) of the \(n\)th demodulated DPSK bit are tabulated in Table 1, together with their probability of occurrence, assuming that ones and zeros are equally probable.

Table 1: Output power of demodulated ASK/DPSK signal

<table>
<thead>
<tr>
<th>ASK</th>
<th>DPSK</th>
<th>Probability of occurrence</th>
<th>Output power from DI</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (a_{n1}=0, a_{n2}=1)</td>
<td>-1</td>
<td>(0)</td>
<td>(P_n=0)</td>
</tr>
<tr>
<td>II (a_{n1}=1, a_{n2}=0)</td>
<td>+1</td>
<td>(\frac{3}{4} \sqrt{P})</td>
<td>(P_n=\frac{1}{4} \sqrt{P} + \frac{1}{4} \sqrt{P})</td>
</tr>
<tr>
<td>III (a_{n1}=0, a_{n2}=0)</td>
<td>(\frac{1}{4} \sqrt{P})</td>
<td>(P_n=\frac{1}{4} \sqrt{P})</td>
<td></td>
</tr>
<tr>
<td>IV (a_{n1}=1, a_{n2}=1)</td>
<td>(\frac{1}{4} \sqrt{P})</td>
<td>(P_n=\frac{1}{4} \sqrt{P})</td>
<td></td>
</tr>
</tbody>
</table>

Assuming Gaussian noise statistics, the error probability of the demodulated DPSK signal is:

\[
prob_{\text{err}} = \text{prob}(0) \cdot \text{prob}(1 | 0) + \text{prob}(1) \cdot \text{prob}(0 | 1)
\]

\[
= \left( \frac{1}{2} - \frac{1}{2} \frac{T_d}{4} \right)^2 \cdot \frac{1}{2} \left( \frac{1}{2} - \frac{1}{2} \frac{T_d}{4} \right)^2 \cdot \text{erfc} \left( \frac{I_0 - R \sigma}{2 \sigma^2} \right) + \frac{1}{2} \frac{T_d}{4} \cdot \frac{1}{2} \left( \frac{1}{2} - \frac{1}{2} \frac{T_d}{4} \right)^2 \cdot \text{erfc} \left( \frac{I_0 - R \sigma}{2 \sigma^2} \right)
\]

\[
+ \frac{1}{2} \frac{T_d}{4} \cdot \frac{1}{2} \left( \frac{1}{2} - \frac{1}{2} \frac{T_d}{4} \right)^2 \cdot \text{erfc} \left( \frac{R \sigma - I_0}{2 \sigma^2} \right)
\]

where \(I_0\) is the decision threshold, \(R\) is the photodiode's responsivity, \(\sigma\) (for \(i \in \{1, 2, \ldots, 5\}\)) is the
receiver noise in each case, and erfc[.] is the complementary error function.

In most applications of optical orthogonal ASK/DPSK signals [1,2], the payload has a few times higher speed than the supervisory/label data. For instance, DPSK payload at 10-Gb/s and ASK label at 2.5-Gb/s or lower in label switching networks. Thus, in our analysis, we assume \( T_d/T_s \leq 1/4 \). Moreover, with small received power at the receiver, thermal noise dominates, thus we can assume \( \sigma_r \) (for \( i \in \{1, \ldots, N\} \)) = \( \sigma_T \). Besides, for a certain range of the ER values, which is found to be 0.63 in <0.95, or 0.2dB in ER< 2dB at the optimal decision threshold \( I_0 \) obtained below, erfc\( [\{I_0 - RP\}/(2\sigma_r)] \) in Eqn (4) is comparable with erfc\( [\{RP - I_0\}/(2\sigma_r)] \), while erfc\( [\{RP - I_0\}/(2\sigma_r)] \) [31] and erfc\( [\{RP - I_0\}/(2\sigma_r)] \) are smaller than erfc\( [\{RP - I_0\}/(2\sigma_r)] \) by a few orders of magnitude.

Hence, under the above conditions, the first two terms can be combined and the 4th, 5th terms can be neglected in Eqn (4), which can then be simplified as:

\[
prob_{r,d} = \frac{1}{2} \cdot \text{erfc} \left( \frac{I_p}{\sqrt{2\sigma_T}} \right) + \left( \frac{1}{4} \right) \cdot \text{erfc} \left( \frac{\{RP - I_0\}/(2\sigma_T]}{\sqrt{2}} \right)
\]  

(5)

Letting \( \hat{prob}_{r,d} = 0 \), we have found that the optimal value of \( I_0 \) can be approximated by (\( rPR/2 \)), for \( prob_{r,d} \leq 10^{-3} \). At this optimal \( I_0 \), we have

\[
prob_{r,d} = \left( \frac{3}{8} \right) \cdot \text{erfc} \left( \frac{Q_i}{\sqrt{2}} \right) \quad \text{where} \quad Q_i = \frac{\{RP - I_0\}/2}{\sigma_T}
\]  

(6)

For \( prob_{r,d} = 10^{-3} \), \( Q_i \approx 5.95 \). \( PR = 11.9\sigma_T/\sigma_r \).

From the power and the probability of each case in Table 1, the average received power of the demodulated DPSK signal equals \( P_{rec} = (r + 1)/4 \).

Together with Eqn (6), the receiver sensitivity (at \( prob_{a,d} = 10^{-3} \), \( P_{rec,d} \)) of the demodulated DPSK signal is given by

\[
P_{rec,d} \text{(dBm)} = P_{rec,d,\sigma} \text{((dBm))} + 10 \log_{10} \frac{r + 1}{2r}
\]  

(7)

On the other hand, the low-speed ASK data with ER=\( r \) can be detected irrespectively to phase modulation via direct square-law detection. Thus, the receiver sensitivity (at \( prob_{a,d} = 10^{-3} \)), \( P_{rec,a} \), of the ASK signal is given by [3]:

\[
P_{rec,a} \text{(dBm)} = P_{rec,a,\sigma} \text{((dBm))} + 10 \log_{10} \frac{1 + r}{1 - r}
\]  

(8)

Analytical and experimental results

We have performed experimental measurements to verify the analytical results. The DPSK bit rate was chosen at 10 Gb/s and the superimposed ASK data was at bit rates of 1- and 1.25-Gb/s. A PIN photodiode with matching electrical bandwidth was used to directly detect the ASK signals. The receiver sensitivities of 1-Gb/s and 1.25-Gb/s ASK signals with \( r=0 \) were measured to be -26.1 dBm and -25.75 dBm, respectively. Fig. 2 depicts their measured receiver sensitivities at other ER (\( r \)) values (1-Gb/s: symbols ‘+’ and 1.25-Gb/s: symbol ‘x’). By using Eqn (8), the analytical receiver sensitivities were also plotted (1-Gb/s: dotted line, 1.25-Gb/s: dashed line, 2.5-Gb/s: dash-dot line) in the same graph, showing good agreement with the experimental results.

To measure the demodulated DPSK sensitivity, the ASK/DPSK signal was first fed into an optical DI before being detected by a 10-Gb/s PIN diode. With the measured receiver sensitivity of -19.2 dBm for the demodulated 10-Gb/s DPSK signal without any ASK signal (i.e. \( r=1 \)), the solid curve was plotted in Fig. 2, by using Eqn (7). The experimental sensitivities were also shown as the symbols ‘+’, ‘o’ and ‘x’ when the superimposed ASK signals were at 2.5-, 1.25- and 1-Gb/s, respectively. The very weak dependence of the demodulated DPSK sensitivity on the ASK bit rate can be understood from the first and the third dominant terms in Eqn (4), i.e. \( prob_{a,d} \) increases a little when \( T_d/T_s \) decreases. However, such trend is not obvious as \( (1/4)(T_d/T_s) \) and \( (1/8)(T_d/T_s) \) are very small.

Conclusion

The signal performance of the ASK/DPSK orthogonal format has been theoretically investigated based on a probability model. The calculations implied an analytical relationship between the ASK/DPSK receiver sensitivities and the extinction ratio as well as the bit rate of the ASK signal, which agreed well with the experimental results.

References